A Novel Meron-induced Pseudospin Wave in Bilayer Quantum Hall Coherent State and the Residual Zero-bias Peak in Tunneling Conductance

Yue Yu
Institute of Theoretical Physics, Chinese Academy of Sciences, P. O. Box 2735, Beijing 100080, China

In the bilayer quantum Hall coherent state for ν_T deviating slightly from one, we show that, instead of the global order parameter, the spontaneous breaking of the pseudospin $U(1)$ rotational symmetry is reflected by the periodic domain structure accompanying with the charged meron pairs. The motion of meron pairs induces a novel pseudospin wave. The long range order of the periodic domains in a low bias voltage range leads to the residual zero-bias peak in the tunneling conductance even when the pseudopsin Goldstone feature in a high bias voltage range can be distinct from it.

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The low-lying gaped and gapless excitations in the various quantum Hall systems played important roles in understanding the essential physics of these systems (For review, see [1,2]). Among them, the most remarkable one was the Laughlin quasiparticle, which has a fractional charge and fractional statistics [3]. The magnetoroton revealed the similarity between the Laughlin liquid and the ^{4}He superfluid [4]. The gapless edge fluctuations distinguished the quantum Hall states from an ordinary insulator [5] and showed a Luttinger liquid behavior [6,7]. The particle-hole continuum of the composite Fermi liquid caused the anomalous propagation of the surface acoustic wave at $\nu = 1/2$ [8,9]. The skyrmion spin texture exhibited a fruitful spin structure in the multi-component quantum Hall systems [10]. The binding-unbinding of the meron-pairs in the bilayer quantum Hall system gave the first example of the finite temperature phase transition in the quantum Hall systems [11,12].

Recently, a pseudospin collective mode [13] in the bilayer spontaneous quantum Hall coherent state at $\nu_T = 1$ has been observed experimentally [14,15]. This can be understood as a pseudospin Goldstone mode due to the spontaneous breaking of symmetry of particle number difference between two layers [16,13]. Accompanying with this linear dispersing collective mode, there existed a Josephson-like tunneling between layers. These intriguing experiments have renewed the theoretical research interest greatly [17–28] while opening a variety of unsolved issues (for a short review, see [29]).

Among these issues, the most urgent two are: First, differing from the Josephson effect in a superconductor junction, the zero-bias conductance peak in the experiment has a finite width and height even at the zero-temperature. Second, in the common Josephson effect, the position of the tunneling peak moves as a magnetic field perpendicular to the tunneling current is applied. In the bilayer quantum Hall case, this Goldstone feature did appear as a parallel field is applied while the central peak remains unexpectedly [15]. In this Letter, we shall focus on the second issue. We see that a new excitation induced by the motion of meron pairs may cause this phenomenon.

Where does this new excitation come from? In realistic samples, there are long-range density fluctuations with about the relative magnitude of 4% as mentioned in [20]. This corresponds to charged meron pair excitations such that the total filling factor ν_T can deviate from one. When the tunneling is turned on, beside the logarithmic interaction between paired merons, there is a linear string confining energy. When the tunneling is small enough, the logarithmic one dominates and determines the optimal separation between two merons constituting a pair. The linear term may distort the pesudospins.(Hereafter we equal the word 'spin' to the pseudospin.) In the picture of [11], only the spins near the string are distorted while the spins far from the string array in the same direction. As we shall see in this Letter, a more realistic spin configuration induced by the meron pairs is a periodic varying spin configuration. Since the $U(1)$ symmetry is spontaneously broken, this periodic configuration will want to lie on a preferred direction. This leads to a long range order of the periodic spin configuration. When the meron pair moves, the spin configuration travels simultaneously. If all charged meron pairs drift in a bias voltage, the motion of the spin configuration forms a spin wave.

Why does this induced spin wave indicate a zero-bias peak in the tunneling conductance? When the meron pairs drift, the induced spin wave travels in a wave velocity $v_{isw} \leq v_m$, the meron drift velocity which is proportional to the bias voltage V. This spin wave contributes to the tunneling current response function a pole $eV \pm eA|V|$ for a constant A, which means a zero-bias peak in the conductance.

Why can the Goldstone mode be seen while this zero-bias peak still exists? In very low bias, only the meron induced spin wave contributes to the tunneling current response function. Since the parallel field can only change the wave length λ of the induced spin wave but does not change the drift velocity of the meron pairs, the zero-bias pole contributed by the induced spin wave was not shifted. As the bias voltage increases, the meron pairs are accelerated. If the meron pairs move so fast that the spin configuration can not respond, the global order parameter is restored because the spins which are not right beside the meron pair can not see the pair. Thus, the

Goldstone mode recovers in the relative high bias, which contributes to the response function. When a parallel field $B_\parallel$ is turned on, the pole of the Goldstone mode shifts to $eV \propto B_\parallel$. Namely, we have two energy scales, while the residual peak exists in the low bias, the Goldstone feature is shown in the high bias.

The finite height and width will not be discussed in details. Several scenarios of the disorder source have been provided [19–21] but a microscopic understanding is still lack. In the present model, the induced spin wave may be dissipative since it is much easier for the meron pair scattering to influence the long range order of the periodic spin configuration than a global order parameter. Thus, it is anticipated that the central peak has a finite height and width. The another possibility is the charging effect of the pseudoskyrmion [27].

Why does the height of this residual peak reduce as the parallel field increases? As we shall see, the domain structure of the meron pairs is controlled by a modulus k. For $B_\parallel = 0$, $k_0 > 1$ implies a finite domain period length. As $B_\parallel$ increases, k decreases and eventually, at a critical $\tilde{B}_\parallel$, k is down to 1. This implies the period length tends to the infinity and the induced spin wave is suppressed. Namely, the height of the central peak of the conductance goes down to zero monotonically as $k \to 1$.

In the pseudospin language, the order parameter is a unit vector $\vec{m} = (\cos\varphi, \sin\varphi, m_z)$. The Hamiltonian for $\nu_T = 1$ layer-balanced coherent states when a parallel field exists is given by [2,11]

$$H = \int d^2r \left\{ \frac{1}{2}\rho_s |\nabla\varphi|^2 - \frac{t}{2\pi l_B^2}\cos(\varphi - Qx) \right\}, \tag{1}$$

where ρ_s is the spin stuffness [11]; l_B is the magnetic length; and $Q = \frac{edB_\parallel}{\hbar c}$ with the layer spacing d; the gauge is chosen as $\vec{A}_\parallel = xB_\parallel \hat{z}$; t is the tunneling amplitude. We turn off $B_\parallel$ first. The cheapest energy charge-$\pm e$ excitation is the meron pair with the opposite vorticities and the same charge ($\pm e/2$) [11]. In the absence of the tunneling, the pair is confined by the logarithmic attraction. It is called a skyrmion [10]. When the tunneling is turned on, the skyrmion may be distorted and eventually turns into a meron pair confined by a domain wall. The domain with an infinite length string along the y axis has its optimal form given by $\varphi(\vec{r}) = 2\arcsin[\tanh s]$, where $s = x/\xi$ with $\xi = \sqrt{2\pi l_B^2 \rho_s / t}$ [11]. In fact, the domain structure may have a general form [30]

$$\varphi(\vec{r}) = 2\arcsin[k\mathrm{sn}(s,k)] + \varphi_0, \tag{2}$$

where φ_0 is a constant and sn is the Jacobian *sine-amplitude* elliptic function. It is given by $\mathrm{sn}(s,k) = k^{-1}\mathrm{sn}(ks, k^{-1})$ if the modulus $k > 1$. The period of $\mathrm{sn}(ks, k^{-1})$ is $4K(k^{-1})/k$ for $k > 1$ where $K(k)$ is the first kind complete elliptic integral. When $k \to 1$, $\mathrm{sn}(s,1) = \tanh s$, going back the solution in [11]. The string tension may be calculated by [31]

$$\mathcal{T}_0(k) = -\frac{\rho_s}{R}\int d^2\vec{r}\varphi\nabla^2\varphi = \mathcal{T}_0 I(k), \tag{3}$$

where $I(k) = \frac{k}{2}[2E(k^{-1}) - \pi\sqrt{1-k^{-2}}]$; $E(k)$ is the second kind complete elliptic integral and $\mathcal{T}_0 = 8\rho_s/\xi$. In the limit of $k \to 1$, $I(1) = 1$ and $\mathcal{T}_0(1) = \mathcal{T}_0$. $I(k)$ monotonically decreases as k increases. If $k \to \infty$, $I(k)$ and $\mathcal{T}_0(k) \to 0$. For $k < 1$, $E(k^{-1})$ is imaginary and (3) has no physical meaning and the only meaningful solution is the trivial one (k=0). The optimal separation between merons in a pair is given by $R_{s0} = e^2/8\pi\rho_s\epsilon$ for the skyrmion or by $R_0(k) = \sqrt{e^2/4\mathcal{T}_0(k)}$ for the domain wall [11]. If $R_0 < R_{s0}$, the minimal pair energy is given by

$$E_{\mathrm{pair}}^{min}(k) = \frac{e^2}{4\epsilon R_0(k)} + \mathcal{T}_0(k)R_0(k) = \sqrt{e^2\mathcal{T}_0(k)/\epsilon}. \tag{4}$$

For $R_0 > R_{s0}$, the k-dependent minimal energy of a pair may be approximated by

$$E_{\mathrm{skyr}}^{min}(k) \approx 2\pi\rho_s(1 + \ln R_{s0}/R_{\mathrm{mc}}) + \mathcal{T}_0(k)R_{s0}, \tag{5}$$

where $R_{\mathrm{mc}} \sim l_B$ is the meron core size. Although this infinite wall result may not be quantitatively correct for $\xi > R_{s0}$, it may grab the qualitatively k-dependent behavior. In (4) and (5), the core energy $2E_{\mathrm{mc}}$ has been omitted. And both of them seem to imply that the state $k \to \infty$ is favorable. However, the domain stores the k-dependent energy given by substituting (2) to (1)

$$E_{\mathrm{domain}}(k) = \frac{At}{2\pi l_B^2}(2k^2 - 1). \tag{6}$$

In the realistic samples, the area A that a meron pair occupied is finite. Thus, one has to minimize

$$E_{\mathrm{total}}(k) = E_{\mathrm{skyr/pair}}^{min}(k) + E_{\mathrm{domain}}(k), \tag{7}$$

in $1 \le k < \infty$. The above discussion is valid if the spacing between meron pairs is larger than the separation of two merons in a pair because the interaction between pairs should be negligible. In the real samples, $\rho_s \sim 0.4$K and $n_0 \sim 5.0 \times 10^{10}\mathrm{cm}^{-2}$, a meron pair occupies an area $\sim 72\ l_B^2$; $t \sim 6.0 \times 10^{-7}$(in the unit $e^2/\epsilon l_B$). Using (5) and (7), one has $k_0 = 3.06$. The space period $\lambda = 4\xi K(k_0^{-1})/k_0 \sim 496 l_B$, about 50 times of the meron pair spacing. To use (4), t is restricted to $0.0016 < t < 0.1$. Taking, for example, $t \sim 0.005$, one has $k_0 \sim 1.04$, $R_0 \sim 4.67 l_B$ and $\xi = 2.65 l_B$. The space period $\lambda \sim 27.57 l_B$, 1.8 times of the domain length. In both large and small t cases, we see there are periodic spin configurations which destroys the global order parameter. As we have mentioned, the spontaneous breaking of U(1) symmetry may lead to all these domains extending in the direction (say $\hat{e}$) along which the spatial average of the order parameter field has a maximal value. Furthermore, the continuation of the order parameter field may require all domains connecting smoothly by self-consistently adjusting the position and φ_0 of each pair. This sets up a

long range order of the periodic spin configuration, e.g., $\langle\psi_\uparrow^\dagger(\vec{r})\psi_\downarrow(\vec{r})\rangle \sim e^{i\varphi(r\hat{e})}$ where $\varphi(r\hat{e})$ is periodic along the $\hat{e}$-direction except in the position of the singular merons. However, the larger t case will not be easy to be observed because the number of domains in a period λ is too small to self-consistently adjust the positions of the meron pairs, which costs the Coulomb energy between the pairs.

Now, turn on the parallel field in the y-direction. Rewriting the Hamiltonian (1) by [2,11]

$$H = \int d^2r \left\{ \frac{1}{2}\rho_s[(\partial_x\theta + Q)^2 + (\partial_y\theta)^2] - tn_0\cos\theta \right\}, \quad (8)$$

where the changed variable $\theta = \varphi - Qx$. Since the extra non-constant term is a total divergence, the domain structure of θ is the same as (2) but the favorable lying direction of the walls is fixed in the y-direction [2,11]. Thus, the Goldstone mode is suppressed.In a very low bias, the periodic domain will move simultaneously as the meron pairs drift. This induces a spin wave, which has the wave velocity $v_{isw} \approx v_m$, wave length λ and wave vector $q_{isw} = h/\lambda$. Instead of the suppressed Goldstone mode, this induced spin wave will contribute to the tunneling current response function. As the bias voltage increases, the response of the induced spin wave to the meron motion becomes slow. And eventually, the spin wave can not respond to the merons. Namely, the periodic domain structure disappears and the Goldstone mode is recovered. We can call this the meron unscreening. The unscreening voltage can be determined by the relaxation time τ_1 of the spin wave. However, this relaxation time is different from the relaxation time τ_φ with a disorder source. τ_1 may be thought as a longitudinal relaxation time while $\tau_\varphi < \tau_1$ is the transverse relaxation time [32]. The transverse τ_φ has been microscopically calculated in [24] while there is no a microscopic estimate to τ_1 yet. However, in our case, τ_1 may be longer than τ_φ several orders because τ_φ indicates the time of a perturbed local spin back to the equilibrium state while τ_1 is the response time of the spin wave following the change of equilibrium state. In the sample of Spielman et al used, $\delta_\varphi = \hbar/\tau_\varphi \sim 0.75$K. If we assume $\tau_1 \sim 10^3\tau_\varphi$, the unscreening voltage V^* can be estimated by $v_m(V^*) \sim 10l_B/\tau_1$. We take the meron pair density $n_m \sim 0.04n_0$ and $n_0 = 5.0\times10^{10}$ cm^{-2}; the longitudinal resistivity $\rho_{xx} \sim 1\text{k}\Omega$ and the sample linear size $\sim$ 1mm. Then, $V^* \sim 50\mu$V, coinciding with the bias voltage in which the Goldstone feature appears in the experiment.

The suppression of the spin wave by a parallel field may be understood as follows. For $B_\parallel \neq 0$, the string tension decreases linearly, i.e., $\mathcal{T}(k) = \mathcal{T}_0(k)(1 - B_\parallel/B_\parallel^*)$, where $B_\parallel^*$ is the critical field of the commensurate-incommensurate phase transition [11]. The optimal value of $k = k_0$ decreases as $\mathcal{T}$ goes down, and at $\tilde{B}_\parallel$, $k_0 \to 1$. For $t = 6.0\times10^{-7}$ but $\mathcal{T} = 0.5\mathcal{T}_0$, $k_0 = 2.38$ and $\lambda \sim 667l_B$. $k_0 = 1$ arrives at $\mathcal{T} = 0.001\mathcal{T}_0$ and $\lambda \to \infty$. That is to say, at $B_\parallel = \tilde{B}_\parallel \lesssim B_\parallel^*$, the induced spin wave is destroyed and one has only the Goldstone mode contributes to the response function. Then the central peak disappears. For larger t, e.g., $t = 0.005$ but $\mathcal{T} = 0.005\mathcal{T}_0$, one has $k_0 \to 1$. Furthermore, for a $B_\parallel > \tilde{B}_\parallel$, (4) and (5) become k-independent but (6) favors $k = 0$. This gives $\theta(\vec{r}) = 0$, namely, $\varphi(\vec{r}) = Qx$, the commensurate state. At $B_\parallel = B_\parallel^*$, the string tension vanishes and a commensurate-incommensurate transition appears [11].

Based on the above discussion, we now calculate the tunneling current in two bias ranges. In the low bias range ($V < V^*$), recalling the expansion $\arcsin[\mathrm{sn}(u)] = \frac{\pi u}{2K} + 2\sum_{n=1}^\infty \frac{1}{n}\frac{g^n}{1+g^{2n}}\sin\frac{n\pi u}{K}$ with $g = e^{-\frac{\pi K'}{K}}$, one can decomposite $\theta(\vec{r}) = \theta_m(\vec{r}) + q_{isw}x + \tilde{\theta}(\vec{r})$, where the first term comes from the singular merons and the other two from the domain. It implies that for an infinite wall, the time-dependent $\tilde{\theta}(x \pm v_{isw}t)$ is the solution of the equation of motion $\frac{1}{v_{isw}^2}\partial_t^2\tilde{\theta} - \partial_x^2\tilde{\theta} = 0$. Hence, the effective Lagrangian of the realistic system for $\omega = eV/\hbar$ reads

$$\mathcal{L} = \frac{\rho_s}{2}\left[\frac{1}{v_{isw}^2}(\partial_t\tilde{\theta}(\vec{r},t))^2 - |\nabla\tilde{\theta}(\vec{r},t)|^2\right] - \frac{t}{2\pi l_B^2}\cos(\theta_m(\vec{r},t) + \tilde{\theta}(\vec{r},t) + q_{isw}x - \omega t). \quad (9)$$

This effective theory is the same as the high bias one by the correspondence $v \leftrightarrow v_{isw}$, $Q \leftrightarrow q_{isw}$ and $\varphi \leftrightarrow \tilde{\theta}$ [19]. The tunneling current now can be calculated in a similar manner in the literature [19–21]. To be specified, we take the calculation result of the tunneling current in the version of Balents and Radzihosky [19],

$$J_Q(V) = \frac{N_0}{l_B q_{isw} k_B T}\sum_s s\left|\frac{k_B T}{eV - s\hbar v_{isw} q_{isw}}\right|^{1-\eta}, \quad (10)$$

where N_0 is a constant [33] and $\eta = k_BT/2\pi\rho_s$ is the Kosterlitz-Thouless exponent. In a very low bias, the velocity $v_{isw} \approx v_m \propto |V|$. Eq.(10), then, implies a zero-bias peak in the condcutance. As V increases, the response of the induced spin wave to the meron pair motion becomes slow. And so the wave velocity reduces a factor which is less than one, i.e., $v_{ism} < v_m$. Thus, the current and the conductance reduce. For $V \to V^*$, $v_{isw} \to 0$ and the sum of s in (10) is zero and the current and conductance vanish. The Q-dependence of (10) is included in $q_{isw} = h/\lambda$. For $B_\parallel = \tilde{B}_\parallel$, $k_0 \to 1$ and $q_{isw} \to 0$. Hence, $J_Q(V) \sim O(q_{isw}) \to 0$. This indicates the zero-bias peak in the conductance is totally suppressed when $B_\parallel = \tilde{B}_\parallel$.

If $V > V^*$, only the Goldstone mode contributes to the tunneling current. The tunneling current in this bias range has been calculated by several authors [19–21]. In [19], the effective Lagrangian and the tunneling current are the same as (9) and (10) by the correspondence mentioned in the last paragraph. The phenomena for two bias ranges are sketched in Figure 1 and resemble what were observed by Spielman et al in their experiment [14,15].

In conclusions, we have found, in a low bias, a long range order of the periodic domains and a meron-induced

spin wave instead of a global order parameter and the Goldstone mode, if ν_T slightly deviates from 1. This leads to a residual conductance zero-bias peak. In a high bias, this induced spin wave disappears as the meron pairs are unscreened and the Goldstone feature is recovered. In a critical parallel magnetic field, this spin wave can also be suppressed. This explains the experimental results in [14,15]. We have assumed that the induced spin wave is perfect. However, the real spin wave shape may be distorted severely due to the merons and their scattering. This is one of the reasons for the finite zero-bias peak.

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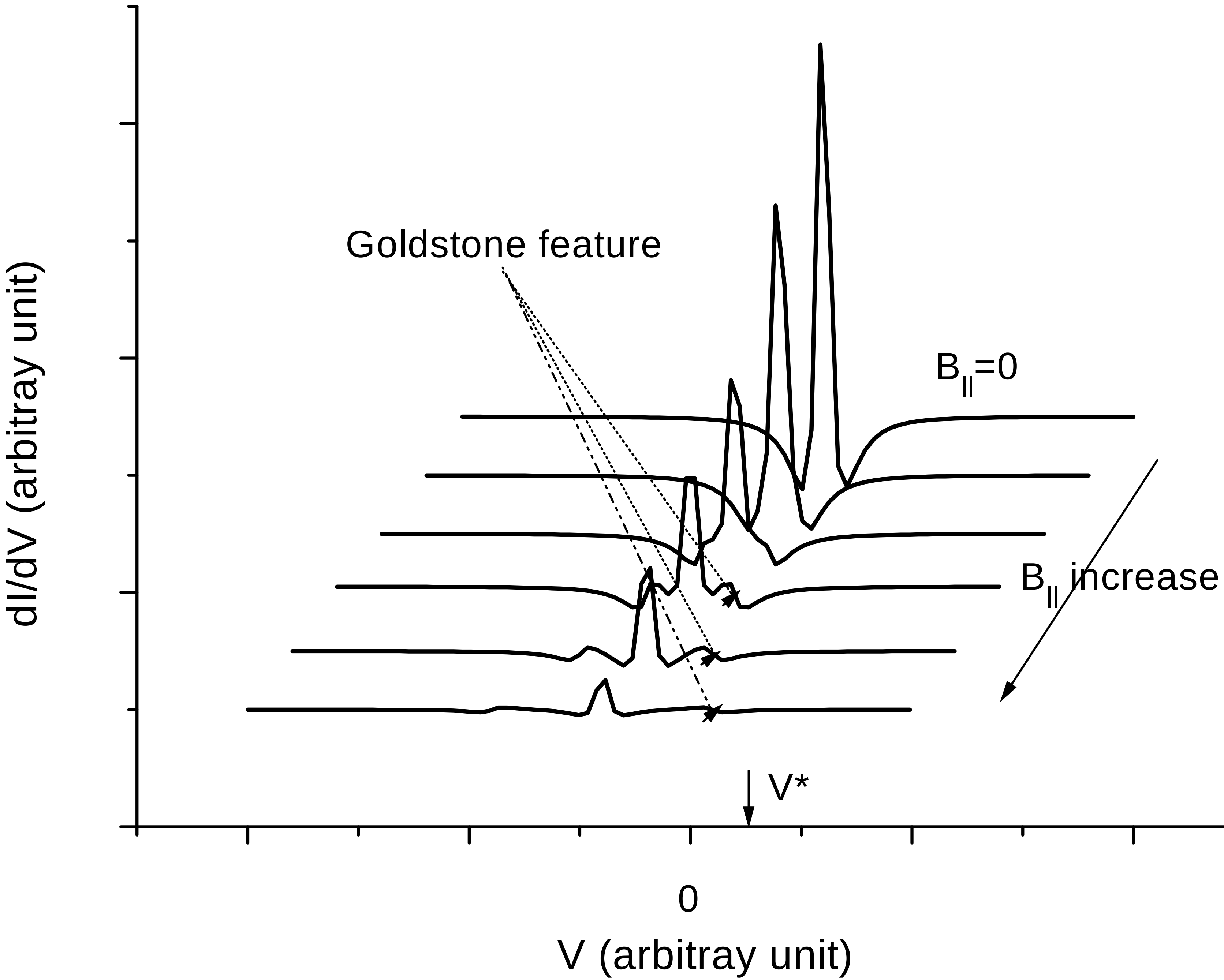
Figure Caption

Fig. 1 Schematic tunneling conductance for $T = 0$ in two bias voltage ranges.The tunneling conductance in $V < V^*$ is given by the derivative of (10). δ_1 and δ_φ ($\delta_1 \ll \delta_\varphi$) have been added in the denominators to round the peak. The Goldstone feature (or the derivative shape) in $V > V^*$ is from the correspondence of (10) [19].


Goldstone feature
$B_{||}$=0
$B_{||}$ increase
V*
0
V (arbitray unit)
dI/dV (arbitray unit)